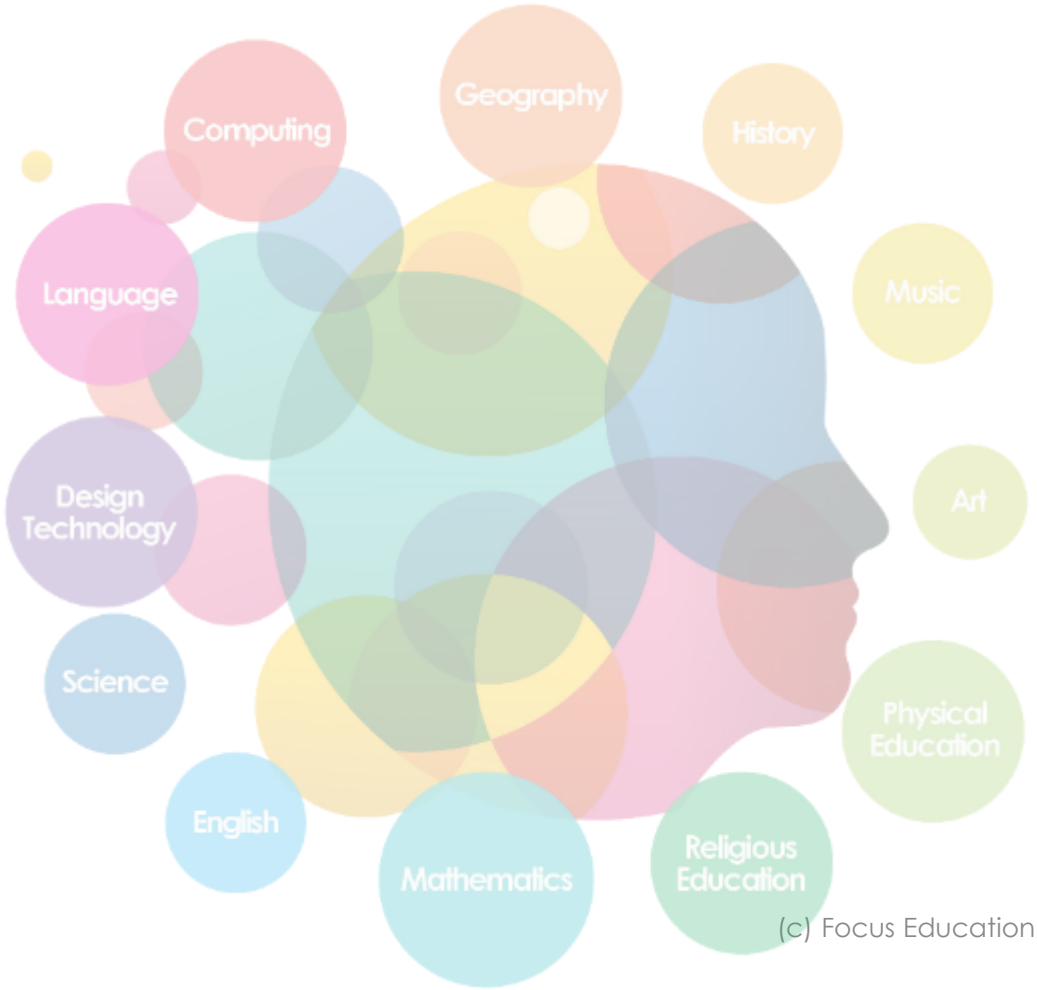
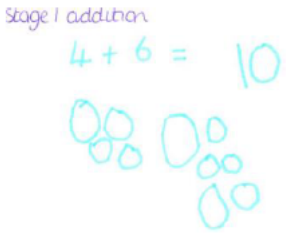
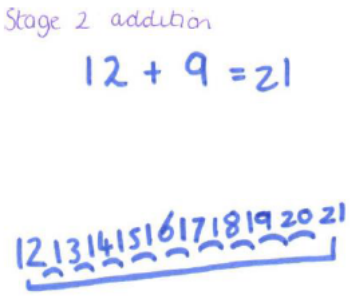
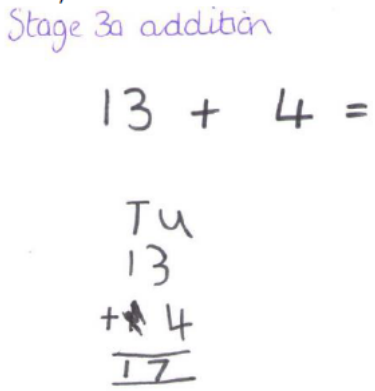
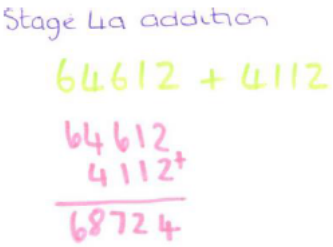
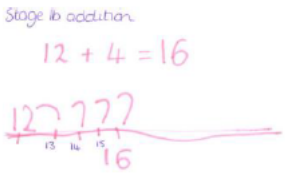
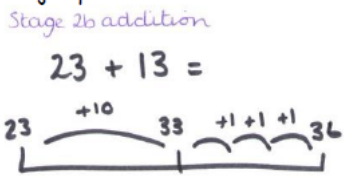
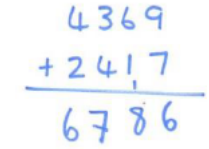
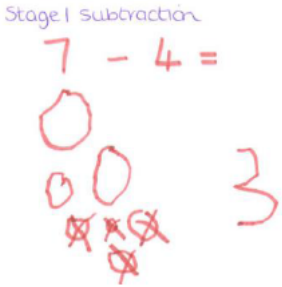
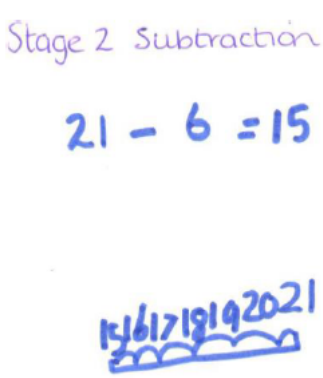
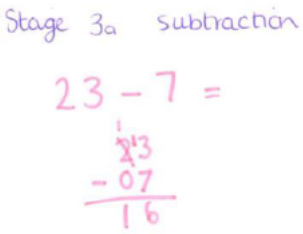
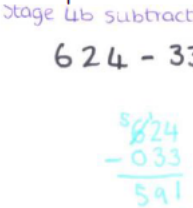
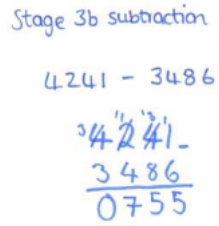
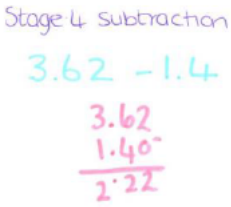
Maths Methods



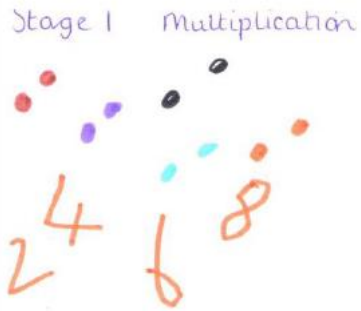
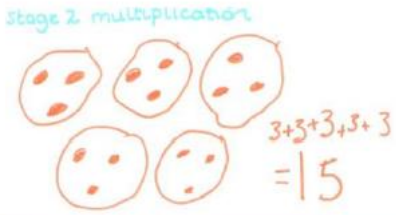
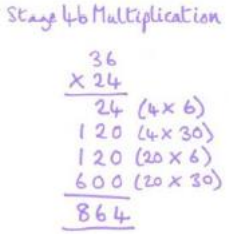
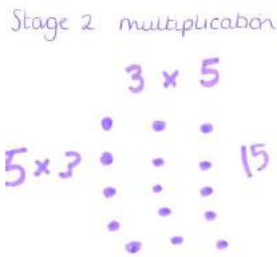
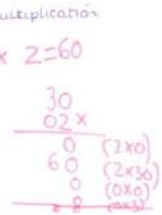
Methods for addition

Stage 1	**Stage 2**	**Stage 3**	**Stage 4**
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.  	Children will begin to use 'empty number lines' themselves starting with the larger number and counting on. • Counting on in tens and ones.  	Adding the least significant digits first,  	Children should extend the carrying method to numbers with at least four digits. 
Children then begin to use numbered lines to support their own calculations.  	• Followed by adding the tens in one jump and the units in one jump.  	• Extending to the short method with carrying over. Note carried digits shown above the line.  	Begin to add two or more decimal fractions with up to four digits and either one or two decimal places.
Children need to be able to: • Recall addition to 20	Children need to be able to: • Partition numbers • Mentally add multiples of 10, 100.	Children need to be able to: • Mentally add larger numbers	
Expected by the end of Reception/Yr1	Expectation by the end of Key Stage 1	Expectation by end of lower KS2	Expectation by end of upper KS2

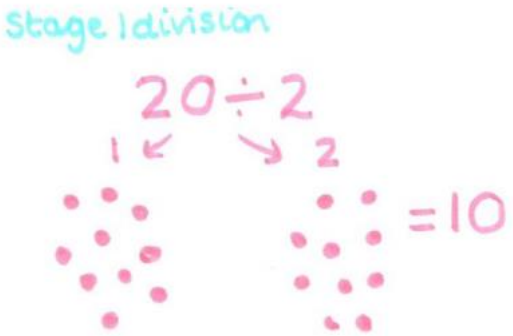
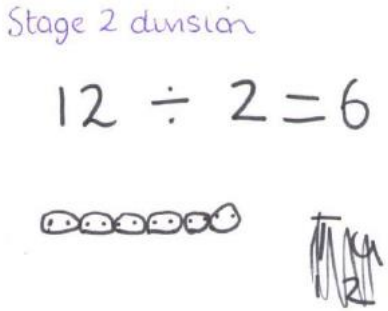
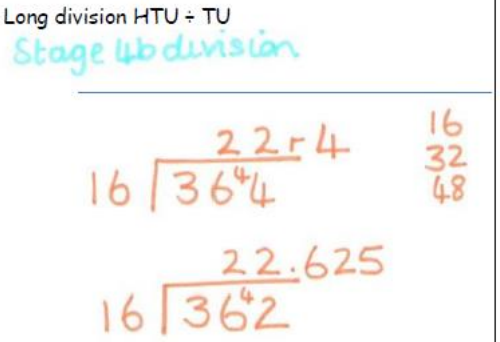
Methods for subtraction

Stage 1	**Stage 2**	**Stage 3**	**Stage 4**
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc. 	Children will begin to use empty number lines to support calculations. • Counting back • First counting back in tens and ones. • Subtracting the tens in one jump and the units in one jump. • Bridging through ten can help children become more efficient. 	Decomposition up to TU and TU  Decomposition from HTU and TU  Decomposition from ThHTU 	Consolidation of subtraction with whole numbers Decomposition with decimals 
Children need to be able to: • Recall addition and subtraction facts to 20 • Subtract multiples of 10 • Know all complements to 10 and 10	Children need to be able to: • Partition 2 and 3 digit numbers • Subtract mentally a single digit number from a 2 digit number	Children need to be able to: • Subtract the totals mentally • Partition numbers mentally	Children need to be able to: • Add the totals mentally • Partition numbers mentally
Expectation by the end of Reception	Expectation by the end of Key Stage 1	Expectation by the end of lower KS2	Expectation by the end of upper KS2

Methods for Multiplication

Stage 1	Stage 2	Stage 3	Stage 4
Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.  	Children will develop their understanding of multiplication and use jottings to support calculation: • Repeated addition  	Children will continue to use: • Repeated addition 4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4 • Arrays As seen previously	TU x TU (Long multiplication – multiplication by more than a single digit) 
	• Arrays Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.  	TU x U (Short multiplication – multiplication by a single digit)  	ThHTU x U (Short multiplication – multiplication by a single digit) HTU x U (Long multiplication – multiplication by more than a single digit) 
Children need to be able to: • Count in steps	Children need to be able to: • Understand multiplication as repeated addition.	HTU x U (Short multiplication – multiplication by a single digit) Children need to be able to: • Recall multiplication facts 12 x 12 • Work out products such as 70 x 5 • Add combinations of numbers mentally	And move onto decimals, For example: 4.92 x 3 
Expectation by end of Reception	Expectation by end of Key Stage 1	Expectation by end of lower KS2	Expectation by end of upper KS2

Methods for Division

Stage 1	Stage 2	Stage 3	Stage 4
Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s. 	Children will develop their understanding of division and use jottings to support calculation • Sharing equally • Grouping or repeated subtraction 	Short division TU ÷ U 	Short division HTU ÷ U  Long division HTU ÷ TU 
Children need to be able to: • Understand division as grouping and sharing (developing to grouping).	Children need to be able to: • Understand multiplication and division as the inverse. • Use multiples of 1, 2, 5, 10 and 20 to derive facts.	Children need to be able to: • Use known facts • Recall multiplication facts 12 x 12 and to understand the inverse.	Children need to be able to: • Use with the most able children who have a secure understanding of all the previous steps.
Expectation by end of Reception	Expectation by end of Key Stage 1	Expectation by end of lower KS2	Expectation by end of upper KS2

Simple progression in written methods for addition

Children need to be able to:
- Recall addition pairs to 9 + 9.
- Know all number bonds to 10.
- Add 3 single-digit numbers, mentally.
- Count on in 1s, 10s and 100s.
- Partition numbers effectively in order to bridge through 10 and 100 eg. 78 + 6 = 78 + 2 + 4

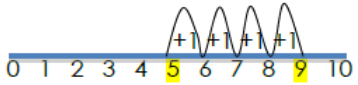
Children need to be able to:
- Partition numbers into hundreds, tens and ones
- Add multiples of 10 or 100 (such as 60 + 70 or 600 + 700)
- Mentally add multiples of 100, 10 and 1 e.g. 200 + 20 + 6, 800 + 130 + 12

Stage 1: Number line (Rec/Yr1)

The numbered line should be used after lots of practical work on addition.

Example:

5 + 4 = 9



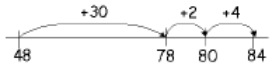
This could progress to counting in 2, 5, 10's.

Stage 1 (cont.): Number line (Rec/Yr1)

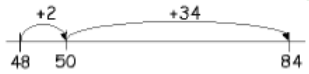
The empty number line helps to record the steps on the way to calculating the total. The steps often bridge through a multiple of 10.

Example:

48 + 36 = 84



or:



Stage 2: Expanded column method (Yr 2/3)

The expanded method leads children to the more compact column method so that they understand the structure and efficiency of it.

The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Children in Year Two should first get used to setting out addition calculations vertically, e.g.

7	12
+ 8	+ 4

Then progress to:
Write the numbers in columns:

Add the ones first
47
+ 76
13
110
123

Discuss how adding the ones first gives the same answer as adding the tens first. Refine over time to consistently adding the ones digits first.
The addition of the tens in the calculation 47 + 76 is described as 'Forty plus seventy equals one hundred and ten', stressing the link to the related fact 'Four plus seven equals eleven'.

Stage 3: Column method (Yr 3/4)

The method is then shortened and when the column total is a two-digit number, the tens (or hundreds) are carried over into the next column. Use the words 'carry ten' or 'carry one hundred', **not** 'carry one'.

Example:

	366
+	458
	824
	11

Once learned, this method is quick and reliable.

Later, **extend** to adding three two-digit numbers, two 4-digit numbers, and numbers with different numbers of digits. This method can also be used to add decimals. **(Yr 5/6)**

Number lines do not always need to be horizontal.

Ask the children to try it both ways: 47 + 76 and 76 + 47. Does it make a difference? Presentation important to accuracy: one number in each square/space/ use of a ruler.

Carry below the line.

Simple progression in written methods for subtraction

Children need to be able to:
- Recall addition pairs to 20
- Add 3 single digit numbers such as 5 + 8 + 4
- Count on in 1s, 2s, 5s and 10s

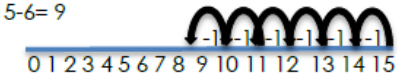
Children need to be able to:
- Partition two-digit and three-digit numbers into multiples of one hundred, ten and one
- Partition numbers in different ways. e.g. 74 into 70 + 4 or 60 + 14
- Subtract mentally a single-digit number or a multiple of 10 or from a two-digit number
- Add the totals (of the hundreds, tens and ones columns) mentally
- Use inverse to check
- Recognise place value of 3 digit number

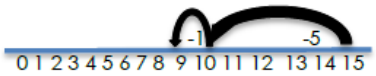
Stage 1: Drawing moving to Number line.

In N/R early subtraction should involve practical activities/ visualising and drawing number of objects and subtracting by crossing out/removing.
The number line needs to initially have pre-printed numbers on; these can be reduced gradually to eventually having an empty number line.

Find the difference by counting back:

15-6= 9





Once the number being subtracted exceeds 10 move onto counting on method:
22-15= 7

The steps may be recorded in a different order or combined. With practice children will record with less information and decide whether to count on for use with larger numbers and in what size of jumps. Each size needs to be modeled.

Stage 2: Vertical with partitioning (Y2 - Y3)

*Continue to use a number line where numbers are close together, e.g. 2004 – 1995

54-23=31

T	U
50	4
20	3
30	1

Year 3 on with exchanging (3d – 1d, 3d -10, 3d-100)
Set calculation out as below. Partition the HTU.
254 – 126= 128

	H	T	U
		40	1
Expand:	200	~~50~~	4
-	100	20	6
	100	20	8

Reinforce place value,
mental partitioning,
Recording larger number first
No use of the + symbol between digits,
Model the order of numbers= makes a difference with subtraction unlike addition. Presentation important to accuracy, so numbers need to line up when necessary.

Stage 3: Decomposition (Y4/5 onwards)

3d-3d up to 1 decimal place

$$\begin{array}{r} {\scriptstyle 4\ \ 1} \\ 2\ \cancel{5}\ 4 \\ -\ 1\ 2\ 6 \\ \hline 1\ 2\ 8 \end{array}$$

Say, "50 – 20" or, "5 tens – 2 tens" not, "5 – 2"

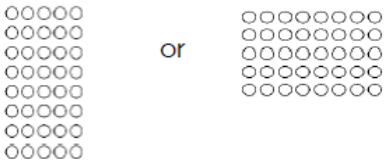
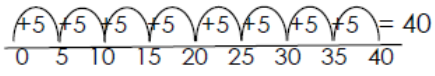
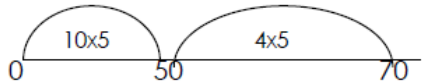
4d -4d up to 2DP

$$\begin{array}{r} {\scriptstyle 5\ \ 1\ \ 1} \\ -\ 3\ \cancel{6}\ \cancel{2}\ 4 \\ 2\ 5\ 7\ 9 \\ \hline 1\ 0\ 4\ 5 \end{array}$$

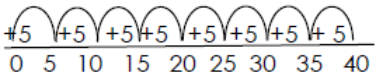
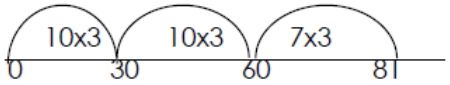
Say, "110 – 70" not "11 – 7"
Say, "500 – 500" not "5 – 5"

Incorporate even more problem solving by Y5/6

Progression in written methods for multiplication

Children need to be able to: 2, 5, 10 X Tables by end of Y2 and inverse division facts. Count in steps of various sizes(1,2,5,10) and others if able.	Children need to be able to: Partition 3d numbers, recall all facts up to 10x10, Find times facts using multiples of 10's(Y4,5,6) Multiply by 10 & 100. Mentally add multiples of 10 & 100.	Children need to be able to multiply by 10 and 100 accurately mentally.
Stage 1-Repeated addition (Rec-Y2) Children start by understanding multiplication as arrays and repeated addition. They use this understanding to help them work out multiplication facts they cannot recall quickly Example: For '8 x 5', children picture   14x5=70  Notes: Blank number line supports learning times tables and using known facts.	**Stage 2-Grid Method (Y2/3)** 25 X 5 = 120 | X	5
---	---
20	100
5	25 | 125 2d X 2d= 24 x 38= 912 | X	20	4
30	600	120
8	160	32 | 720 + 192 = 912	**Stage 3-Short (Y3/4) leading to long (Y5/6) Multiplication 4 digit by 2 digit.** 25 X 5 = 120 25 → 25 X 5 X 5 25 125 100 2 **125** 1 3 24 X 38 192 720 **912** **1**
They use repeated addition to workout the calculation. Recording of the steps on the number line.		Notes: When introducing short multiplication teach using grid and short to show the link.

Progression in written methods for Division

Children need to: know that division equals grouping/sharing, that division is the inverse of multiplication, know X facts to 10 x 10, derive known facts e.g., 10x3=30 so 20 x 3=60, proficient at subtraction on a number line, add multiples mentally, e.g. 70+70.	Children need to be able to: Count on from 0 in 2d numbers.	Children need to be able to multiply by 10 and 100 accurately mentally and have a fluency in all their tables and use of multiples.
Stage 1: Sharing/grouping-(EYFS/Y1) 8/2=4 OOOO OOOO **Stage 2: Repeated addition-from Y1** 40/5=8  **Y2/3 (larger numbers & use this method with remainders)** 81/3= 21  	**Stage 3-Short division(Y3/4)** 2 7 3 | 8 1 3 6 9 12 15 18 21	**Stage 4-Short division with 2d divisor Y5/6** 384/16=24 0 2 4 16 | 3 8 6 4 16 32 48 64 80 Pupils progress to 4d divide by 2 d 2436/16=152r4 0 1 5 2 r4 16 | 2 4 3 6 (8 3) This will lead to pupils developing knowledge of remainders and converting into a fraction and then a decimal. 152 r4 becomes 152 ¼= 152.25